

1 The circle C has the equation $x^2 + y^2 - 2x + 6y = 26$

Find:

- (i) The coordinates of the centre of C
- (ii) the radius of C

$$x^2 - 2x + y^2 + 6y = 26$$

$$(x - 1)^2 - 1 + (y + 3)^2 - 9 = 26$$

$$(x - 1)^2 + (y + 3)^2 = 36$$

i) (1, -3)

ii) 6

2 The circle C has centre $(2, 5)$ and passes through point $(4, 9)$.

Find an equation for C .

$$(x - 2)^2 + (y - 5)^2 = r^2$$

$$(4 - 2)^2 + (9 - 5)^2 = r^2$$

$$20 = r^2$$

$$\underline{\underline{(x - 2)^2 + (y - 5)^2 = 20}}$$

3 The circle C has centre $(-2, 3)$ and passes through point $(1, 8)$.

(a) Find an equation for C .

(4)

(b) Show that the point $(3, 6)$ lies on C .

(1)

(c) Find the equation to the tangent to C at $(3, 6)$.

(5)

Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.

a/
$$(x + 2)^2 + (y - 3)^2 = r^2$$

$$(1 + 2)^2 + (8 - 3)^2 = r^2$$

$$34 = r^2$$

$$(x + 2)^2 + (y - 3)^2 = 34$$

b/
$$(3 + 2)^2 + (6 - 3)^2 = 34$$

$$34 = 34 \quad \checkmark$$

c/
$$\begin{array}{cc} (-2, 3) & (3, 6) \\ x_1, y_1 & x_2, y_2 \end{array}$$

$$\text{gradient of radius} = \frac{6 - 3}{3 - (-2)} = \frac{3}{5}$$

tangent is perpendicular $\therefore m = -\frac{5}{3}$ $\begin{pmatrix} 3, 6 \\ x_1, y_1 \end{pmatrix}$

$$y - 6 = -\frac{5}{3}(x - 3)$$

$$3(y - 6) = -5(x - 3)$$

$$3y - 18 = -5x + 15$$

$$\underline{\underline{5x + 3y - 33 = 0}}$$

4 The circle C has centre $(2, 5)$ and radius 7.

(a) Find an equation for C .

(2)

The line $y = 3x - 1$ intersects C at the points A and B .

(b) Find the exact coordinates of A and B .

(6)

a/
$$(x - 2)^2 + (y - 5)^2 = 49$$

b/
$$(x - 2)^2 + (3x - 1 - 5)^2 = 49$$

$$(x - 2)^2 + (3x - 6)^2 = 49$$

$$x^2 - 2x - 2x + 4 + 9x^2 - 18x - 18x + 36 = 49$$

$$10x^2 - 40x - 9 = 0$$

$$x = \frac{20 + 7\sqrt{10}}{10}$$

$$x = \frac{20 - 7\sqrt{10}}{10}$$

$$y = \frac{50 + 21\sqrt{10}}{10}$$

$$y = \frac{50 - 21\sqrt{10}}{10}$$

$$\left(\frac{20 + 7\sqrt{10}}{10}, \frac{50 + 21\sqrt{10}}{10} \right) \text{ and } \left(\frac{20 - 7\sqrt{10}}{10}, \frac{50 - 21\sqrt{10}}{10} \right)$$

- 5 The circle C has the equation $x^2 + y^2 + 8x - 4y + k = 0$
Where k is a constant.

Given that the point $(1, 5)$ lies on C .

(a) Find the value of k (2)

(b) Find the coordinates of the centre and the radius of C (3)

A straight line that passes through the point $A(3, 7)$ is a tangent to the circle C at the point B

(c) Find the exact length of the line AB (5)

a/

$$(1)^2 + (5)^2 + 8(1) - 4(5) + k = 0$$

$$14 + k = 0$$

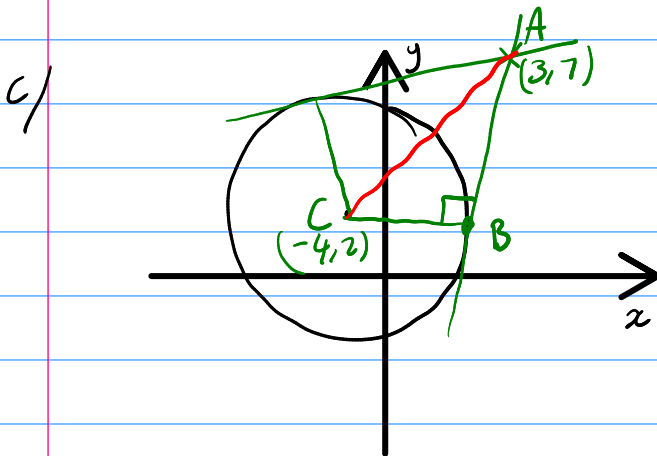
$$\underline{k = -14}$$

b/

$$(x + 4)^2 - 16 + (y - 2)^2 - 4 - 14 = 0$$

$$(x + 4)^2 + (y - 2)^2 = 34$$

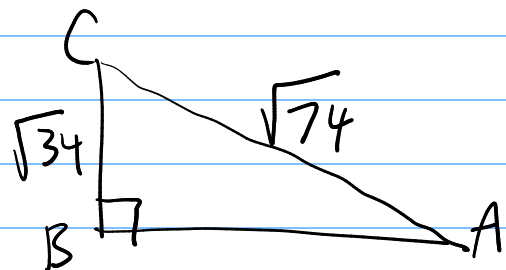
Centre: $(-4, 2)$ radius = $\sqrt{34}$



$$AC \text{ length}^2 = (3 - (-4))^2 + (7 - 2)^2$$

$$AL \text{ length}^2 = 7^2 + 5^2$$

$$\text{length} = \sqrt{74}$$



$$AB^2 = AC^2 - BC^2$$

$$\underline{\underline{AB = \sqrt{40}}}$$

6 The points D, E and F have coordinates $(-3, 2), (4, -1)$ and $(1, -8)$ respectively.
 x_1, y_1, x_2, y_2

(a) Show that angle DEF is a right angle.

(4)

Given that D, E and F all lie on the circle C .

(b) Find the coordinates of the centre of C .

(3)

(c) Find the equation of the circle C .

(3)

a/ DE must be perpendicular to EF .

$$DE \quad m = \frac{-1 - 2}{4 - (-3)} = \frac{-3}{7}$$

$$EF \quad m = \frac{-8 - (-1)}{1 - 4} = \frac{-7}{-3} = \frac{7}{3}$$

$$\frac{-3}{7} \times \frac{7}{3} = -1 \quad \therefore \text{perpendicular} \checkmark$$

b/ As DEF is a right angle DF must be a diameter.

Centre is midpoint of DF .

$$\left(\frac{-3 + 1}{2}, \frac{2 - 8}{2} \right)$$

$$\underline{\underline{(-1, -3)}}$$

c/ $(x + 1)^2 + (y + 3)^2 = r^2$

$$(-3 + 1)^2 + (2 + 3)^2 = r^2$$

$$r^2 = 29$$

$$\underline{\underline{(x + 1)^2 + (y + 3)^2 = 29}}$$

7 The circle C has the equation $x^2 + y^2 - 6x + 2y = 6$

(a) Find the coordinates of the centre and the radius of C

(3)

C crosses the y axis at the points A and B

(b) Find the coordinates of the points A and B

(3)

a) $x^2 - 6x + y^2 + 2y = 6$

$$(x - 3)^2 - 9 + (y + 1)^2 - 1 = 6$$

$$(x - 3)^2 + (y + 1)^2 = 16$$

centre $(3, -1)$ radius $= 4$

b) crosses y when $x = 0$

$$y^2 + 2y = 6$$

$$y^2 + 2y - 6 = 0$$

$$\underline{y = -1 + \sqrt{7}} \quad \text{or} \quad \underline{y = -1 - \sqrt{7}}$$

$$\underline{(0, -1 + \sqrt{7})}$$

$$\underline{(0, -1 - \sqrt{7})}$$

8 The points A and B have coordinates $(-3, 5)$ and $(13, -4)$ respectively.

Given that AB is a diameter of the circle C .

Find an equation for C .

$$\text{centre } \left(\frac{-3+13}{2}, \frac{5-4}{2} \right)$$

$$\left(5, \frac{1}{2} \right)$$

$$(x-5)^2 + \left(y - \frac{1}{2}\right)^2 = r^2$$

$$(-3-5)^2 + \left(5 - \frac{1}{2}\right)^2 = r^2$$

$$r^2 = \frac{337}{4}$$

$$(x-5)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{337}{4}$$

9 The circle C has centre (1, 5) and passes through the point A (-4, 3).

(a) Find an equation for C.

(4)

(b) Find an equation for the tangent to C at A, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers

(4)

$$a/ \quad (x - 1)^2 + (y - 5)^2 = r^2$$

$$(-4 - 1)^2 + (3 - 5)^2 = r^2$$

$$r^2 = 29$$

$$(x - 1)^2 + (y - 5)^2 = 29$$

$$b/ \quad \text{gradient of radius} = \frac{3 - 5}{-4 - 1} = \frac{-2}{-5} = \frac{2}{5}$$

$$\therefore m = -\frac{5}{2}$$

$$y - 3 = -\frac{5}{2}(x + 4)$$

$$2(y - 3) = -5(x + 4)$$

$$2y - 6 = -5x - 20$$

$$\underline{\underline{5x + 2y + 14 = 0}}$$

10 The circle C has centre $(5, k)$, where k is a constant.

The line $y = 2x + 1$ is a tangent to the circle C , touching C at the point $A(3, 7)$.

Find an equation for C .

gradient of radius = $-\frac{1}{2}$ (perpendicular)

$$\frac{7 - k}{3 - 5} = -\frac{1}{2}$$

$$2(7 - k) = -1(-2)$$

$$14 - 2k = 2$$

$$12 = 2k$$

$$\underline{\underline{k = 6}}$$

$$(x - 5)^2 + (y - 6)^2 = r^2$$

$$(3 - 5)^2 + (7 - 6)^2 = r^2$$

$$r^2 = 5$$

$$\underline{\underline{(x - 5)^2 + (y - 6)^2 = 5}}$$

11 The circle C has the equation $x^2 + y^2 - 14x + 2y + 40 = 0$

(a) Find:

- (i) The coordinates of the centre of C
- (ii) the exact radius of C

(3)

The line with equation $y = kx$ where k is a constant, meets C at two distinct points.

(b) Find the range of possible values of k .

(6)

a) $(x - 7)^2 - 49 + (y + 1)^2 - 1 + 40 = 0$

$$(x - 7)^2 + (y + 1)^2 = 10$$

i/ $(7, -1)$

ii/ $\sqrt{10}$

b/ $y = kx$ $x^2 + (kx)^2 - 14x + 2(kx) + 40 = 0$

$$x^2 + k^2x^2 - 14x + 2kx + 40 = 0$$

$$(k^2 + 1)x^2 + (2k - 14)x + 40 = 0$$

2 solutions $\therefore b^2 - 4ac > 0$

$$(2k - 14)^2 - 4(k^2 + 1)(40) > 0$$

$$4k^2 - 28k - 28k + 196 - 160k^2 - 160 > 0$$

$$-156k^2 - 56k + 36 > 0$$

$$\underline{\underline{-\frac{9}{13} < k < \frac{1}{3}}}$$

12 The circle C has the equation $x^2 + y^2 - 8x + 6y + 5 = 0$

(a) Find:

- (i) The coordinates of the centre of C
- (ii) the exact radius of C

(3)

The line with equation $y = k$ where k is a constant, is a tangent to C .

(b) Find the possible values of k .

(2)

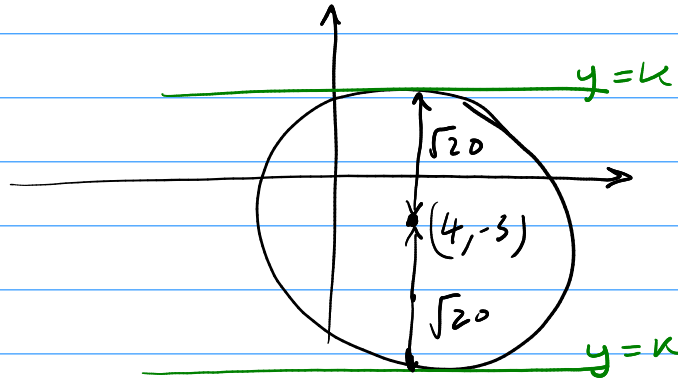
$$(x - 4)^2 - 16 + (y + 3)^2 - 9 + 5 = 0$$

$$(x - 4)^2 + (y + 3)^2 = 20$$

i/ $(4, -3)$

ii/ $\sqrt{20}$

b/



$$\underline{\underline{k = -3 + \sqrt{20}}} \quad \text{or} \quad \underline{\underline{k = -3 - \sqrt{20}}}$$

13 The circle C has the equation $x^2 + y^2 - 6x - 12y = 0$

The line l is a tangent to C at the point $(8, 3)$.

Find the equation of l in the form $ax + by + c = 0$

$$(x - 3)^2 - 9 + (y - 6)^2 - 36 = 0$$

$$(x - 3)^2 + (y - 6)^2 = 45$$

centre $(3, 6)$

$$\text{gradient of radius} = \frac{6 - 3}{3 - 8} = -\frac{3}{5}$$

$$\therefore \text{gradient of tangent} = \frac{5}{3}$$

$$y - 3 = \frac{5}{3}(x - 8)$$

$$3(y - 3) = 5(x - 8)$$

$$3y - 9 = 5x - 40$$

$$0 = \underline{\underline{5x - 3y - 31}}$$

14 The circle C has the equation $x^2 + y^2 - 8x - 10y + k = 0$, where k is a constant.

Given the C lies entirely in the first quadrant, find the range of possible values of k .

$$(x - 4)^2 - 16 + (y - 5)^2 - 25 + k = 0$$

$$(x - 4)^2 + (y - 5)^2 = 41 - k$$

centre $(4, 5)$ radius must be less than 4
 $\therefore r^2 < 16$

$$r^2 = 41 - k \quad (k \text{ must be less than } 41)$$

$$41 - k < 16$$

$$25 < k$$

$$\underline{25 < k < 41}$$

- 15 The line $y = mx - 2$ is a tangent to the circle $x^2 + 6x + y^2 - 8y + 5 = 0$
Find the two possible values of m , giving your answers in exact form.

$$\begin{aligned}x^2 + 6x + (mx - 2)^2 - 8(mx - 2) + 5 &= 0 \\x^2 + 6x + m^2x^2 - 4mx + 4 - 8mx + 16 + 5 &= 0 \\(1 + m^2)x^2 + (6 - 12m)x + 25 &= 0\end{aligned}$$

Tangent has one solution $\therefore b^2 - 4ac = 0$

$$\begin{aligned}(6 - 12m)^2 - 4(1 + m^2)(25) &= 0 \\36 - 72m - 72m + 144m^2 - 100 - 100m^2 &= 0 \\44m^2 - 144m - 64 &= 0\end{aligned}$$

$$m = \frac{18 + 10\sqrt{5}}{11} \quad \text{or} \quad m = \frac{18 - 10\sqrt{5}}{11}$$

- 16 The line $y = mx + 2$ is a tangent to the circle $(x - 5)^2 + (y + 1)^2 = 15$
Find the two possible values of m , giving your answers in exact form.

$$\begin{aligned}(x - 5)^2 + (mx + 2 + 1)^2 &= 15 \\(x - 5)^2 + (mx + 3)^2 &= 15 \\x^2 - 10x + 25 + m^2x^2 + 6mx + 9 &= 15 \\(1 + m^2)x^2 + (6m - 10)x + 19 &= 0\end{aligned}$$

tangent $\therefore b^2 - 4ac = 0$

$$\begin{aligned}(6m - 10)^2 - 4(1 + m^2)(19) &= 0 \\36m^2 - 120m + 100 - 76 - 76m^2 &= 0 \\-40m^2 - 120m + 24 &= 0\end{aligned}$$

$$m = \frac{-15 + \sqrt{285}}{10} \quad \text{or} \quad m = \frac{-15 - \sqrt{285}}{10}$$

17 The circle C has the equation $x^2 + y^2 - 4x - 6y = 48$

(a) Find the coordinates of the centre of C

(2)

(b) Find equation of the tangent to the circle at the point $(7, 9)$

(4)

$$a/ \quad (x-2)^2 - 4 + (y-3)^2 - 9 = 48$$

$$(x-2)^2 + (y-3)^2 = 61$$

(2, 3)

$$b/ \quad \text{gradient of radius} = \frac{9-3}{7-2} = \frac{6}{5}$$

$$\text{gradient of tangent} = -\frac{5}{6}$$

$$y-9 = -\frac{5}{6}(x-7)$$

$$6(y-9) = -5(x-7)$$

$$6y-54 = -5x+35$$

$$\underline{\underline{5x+6y-89=0}}$$

18 A circle C has centre $(10, 6)$ and radius $2\sqrt{17}$

A line L has equation $y = mx$

(a) Show that the x -coordinate of any points of intersection of C and L satisfies the equation.

$$(m^2 + 1)x^2 - (12m + 20)x + 68 = 0 \quad (3)$$

(b) Find values of m for which the equation in (a) has equal roots. (3)

(c) Two lines drawn from the origin which are tangents to C .

Find the coordinates of the points of contact between the tangents and C . (4)

$$r^2 = (2\sqrt{17})^2 \\ = 68$$

$$(x - 10)^2 + (y - 6)^2 = 68$$

$$(x - 10)^2 + (mx - 6)^2 = 68$$

$$x^2 - 10x - 10x + 100 + m^2x^2 - 6mx - 6mx + 36 = 68$$

$$(1 + m^2)x^2 + (-20 - 12m)x + 68 = 0$$

$$(m^2 + 1)x^2 - (12m + 20)x + 68 = 0$$

b/ $b^2 - 4ac = 0$

$$(12m + 20)^2 - 4(m^2 + 1)(68) = 0$$

$$144m^2 + 480m + 400 - 272m^2 - 272 = 0$$

$$-128m^2 + 480m + 128 = 0$$

$$4m^2 - 15m - 4 = 0$$

$$m = 4 \quad \text{or} \quad m = -\frac{1}{4}$$

c/ $y = 4x$ and $y = -\frac{1}{4}x$

when $m = 4$

$$17x^2 - 68x + 68 = 0$$

$$x = 2 \quad y = 8$$

$$\underline{\underline{(2, 8)}}$$

when $m = -\frac{1}{4}$

$$\frac{17}{16}x^2 - 17x + 68 = 0$$

$$x = 8 \quad y = -2$$

$$\underline{\underline{(8, -2)}}$$

- 19 The lines with equations $y = \frac{2}{3}x$ and $y = -\frac{2}{3}x$ are tangents to a circle at $(3, 2)$ and $(3, -2)$ respectively.

x_1, y_1 x_1, y_1

Find an equation for the circle.

radius at $(3, 2)$

$$m = -\frac{3}{2}$$

$$y - 2 = -\frac{3}{2}(x - 3)$$

$$2(y - 2) = -3(x - 3)$$

$$2y - 4 = -3x + 9$$

$$\underline{3x + 2y = 13}$$

radius at $(3, -2)$

$$m = \frac{3}{2}$$

$$y + 2 = \frac{3}{2}(x - 3)$$

$$2y + 4 = 3x - 9$$

$$\underline{13 = 3x - 2y}$$

Intersection is solution to sim. equations

$$x = \frac{13}{3} \quad y = 0$$

$$\left(x - \frac{13}{3}\right)^2 + y^2 = r^2$$

$$\left(3 - \frac{13}{3}\right)^2 + 2^2 = r^2$$

$$r^2 = \frac{52}{9}$$

$$\underline{\underline{\left(x - \frac{13}{3}\right)^2 + y^2 = \frac{52}{9}}}$$

20 The circle with equation $x^2 + y^2 - 10x + ky + 20 = 0$ has radius 3.

Find the two possible values of the constant k .

$$(x - 5)^2 - 25 + \left(y + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 20 = 0$$

$$(x - 5)^2 + \left(y + \frac{k}{2}\right)^2 = \frac{k^2}{4} + 5$$

$$\sqrt{\frac{k^2}{4} + 5} = 3$$

$$\frac{k^2}{4} + 5 = 9$$

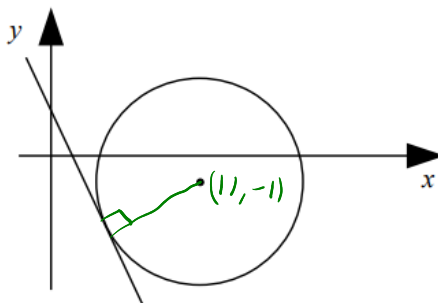
$$\frac{k^2}{4} = 4$$

$$k^2 = 16$$

$$k = \pm 4$$

21 The diagram shows line with equation $y + 3x = 2$ which is a tangent to a circle with centre $(11, -1)$.

$$y = -3x + 2$$



$$(y + 1)^2 + (x - 11)^2 = r^2$$

Find an equation for the circle.

gradient of radius = $\frac{1}{3}$ (perpendicular)

equation of radius:

$$y + 1 = \frac{1}{3}(x - 11)$$

$$3y + 3 = x - 11$$

$$14 = x - 3y$$

$$3x + y = 2$$

intersection $(2, -4)$

$$r^2 = (11 - 2)^2 + (-1 - (-4))^2$$

$$r^2 = 90$$

$$\underline{(x - 11)^2 + (y + 1)^2 = 90}$$

22 Find the centre of the circle with equation $x^2 + y^2 - 8x + 10y = 15$

$$(x - 4)^2 - 16 + (y + 5)^2 - 25 = 15$$

$$\underline{\underline{(4, -5)}}$$

23 The points $A(4, a)$ and $B(13, 6)$ lie on a circle.

AB is a diameter of the circle and has a gradient of $\frac{1}{3}$

The circle has equation $(x - c)^2 + (y - d)^2 = e$ where c, d and e are rational numbers.

Find the values of a, c, d and e .

equation of diameter: $y - 6 = \frac{1}{3}(x - 13)$

$$3y - 18 = x - 13$$

$$3y = x + 5$$

$$\underline{\underline{y = \frac{1}{3}x + \frac{5}{3}}}$$

when $x = 4$ $y = \frac{1}{3}(4) + \frac{5}{3}$
 $\underline{\underline{= 3}}$

centre $\left(\frac{4+13}{2}, \frac{3+6}{2}\right)$

$$\left(\frac{17}{2}, \frac{9}{2}\right)$$

$$\left(x - \frac{17}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = r^2$$

$$\left(13 - \frac{17}{2}\right)^2 + \left(6 - \frac{9}{2}\right)^2 = r^2$$

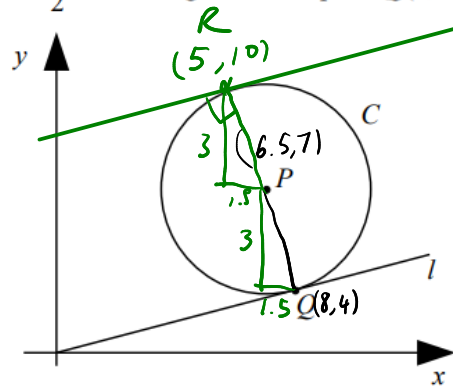
$$r^2 = \frac{45}{2}$$

$$\left(x - \frac{17}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{45}{2}$$

$$a = 3 \quad c = \frac{17}{2} \quad d = \frac{9}{2} \quad e = \frac{45}{2}$$

24 The sketch shows circle C with centre $P(6.5, 7)$

The line l with equation $y = \frac{1}{2}x$ is a tangent to C at point $Q(8, 4)$



(a) Find an equation for C

(4)

The line with equation $y = \frac{1}{2}x + k$, where k is a non-zero constant, is also a tangent to C .

(b) Find the value of k .

(3)

$$a/ \quad (x - 6.5)^2 + (y - 7)^2 = r^2$$

$$r^2 = 1.5^2 + 3^2$$

$$r^2 = \frac{45}{4}$$

$$(x - 6.5)^2 + (y - 7)^2 = \frac{45}{4}$$

b/ passes through $(5, 10)$ $QP = PR$

$$y = \frac{1}{2}x + k$$

$$10 = \frac{1}{2}(5) + k$$

$$10 = \frac{5}{2} + k$$

$$k = \frac{15}{2}$$

25 A circle has equation $(x + 3)^2 + (y - 4)^2 = 25$

Find the gradient of the tangent to the circle at the origin.

$$(0, 0) \quad (-3, 4)$$

$$\text{gradient of radius} = \frac{4}{-3} = -\frac{4}{3}$$

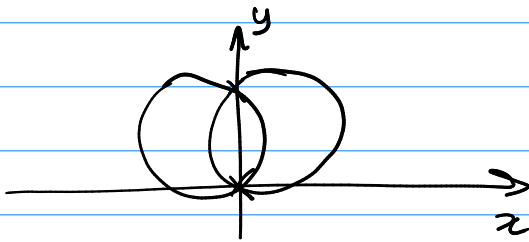
$$\therefore \text{gradient of tangent} = \frac{3}{4}$$

26 A circle C with radius 5 passes through $(0, 0)$ and $(0, 8)$.

(a) Sketch the two possible positions of C. (1)

(b) Find the equations of the two circles. (3)

a/



b/ centre has y coordinate 4

$$(x - a)^2 + (y - 4)^2 = 25$$

$$(-a)^2 + (-4)^2 = 25$$

$$a^2 + 16 = 25$$

$$a^2 = 9$$

$$a = \pm 3$$

$$\text{and } (x - 3)^2 + (y - 4)^2 = 25$$

$$\text{and } (x + 3)^2 + (y - 4)^2 = 25$$

27 A circle C with centre $(2, 1)$ passes through the point $(-1, 3)$.

A line L passes through the points $(1, -4)$ and $(10, 2)$.

Show that L is a tangent to C .

$$\begin{aligned}C: \quad (x-2)^2 + (y-1)^2 &= r^2 \\ (-1-2)^2 + (3-1)^2 &= r^2 \\ 13 &= r^2 \\ (x-2)^2 + (y-1)^2 &= 13\end{aligned}$$

$$L: \quad m = \frac{2 - (-4)}{10 - 1} = \frac{6}{9} = \frac{2}{3}$$

$$y - 2 = \frac{2}{3}(x - 10)$$

$$3(y - 2) = 2(x - 10)$$

$$3y - 6 = 2x - 20$$

$$3y = 2x - 14$$

$$y = \frac{2}{3}x - \frac{14}{3}$$

L and C intersect where

$$(x-2)^2 + \left(\frac{2}{3}x - \frac{14}{3} - 1\right)^2 = 13$$

$$x^2 - 4x + 4 + \left(\frac{2}{3}x - \frac{17}{3}\right)^2 = 13$$

$$x^2 - 4x + 4 + \frac{4}{9}x^2 - \frac{68}{9}x + \frac{289}{9} = 13$$

$$9x^2 - 36x + 36 + 4x^2 - 68x + 289 = 117$$

$$13x^2 - 104x + 208 = 0$$

$$x^2 - 8x + 16 = 0$$

$$(x-4)^2 = 0$$

$$\underline{\underline{x = 4}}$$

one solution

\therefore tangent

at $(4, -2)$

28 A circle C with centre $(4, -1)$ and radius 10.

A line L passes through the points $(-2, 1)$ and $(14, 9)$.

Find the points of intersection of L and C .

$$C: (x - 4)^2 + (y + 1)^2 = 100$$

$$L: m = \frac{9 - 1}{14 - (-2)} = \frac{8}{16} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x + 2)$$

$$y - 1 = \frac{1}{2}x + 1$$

$$y = \frac{1}{2}x + 2$$

Intersection where:

$$(x - 4)^2 + \left(\frac{1}{2}x + 2 + 1\right)^2 = 100$$

$$(x - 4)^2 + \left(\frac{1}{2}x + 3\right)^2 = 100$$

$$x^2 - 8x + 16 + \frac{1}{4}x^2 + 3x + 9 = 100$$

$$\frac{5}{4}x^2 - 5x + 25 = 100$$

$$\frac{5}{4}x^2 - 5x - 75 = 0$$

$$x^2 - 4x - 60 = 0$$

$$(x - 10)(x + 6) = 0$$

$$x = 10 \quad x = -6$$

$$y = \frac{1}{2}(10) + 2 \quad y = \frac{1}{2}(-6) + 2$$
$$= 7 \quad = -1$$

$$\underline{(10, 7)} \quad \text{and} \quad \underline{(-6, -1)}$$